Planck's Constant found encoded in the Great Pyramid of Giza

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ABSTRACT:

Upon computing the total energy required to raise all the stones to their respective heights in the Great Pyramid, and dividing by the number of stone steps in the construction, we obtain the energy of a photon that has the wavelength of the Planck Length. This observation enables us to calculate an estimate of Planck's Constant, h, from the measurements of the Great Pyramid of Giza, and provides a rationale for the architect's particular choice of the number of stone step layers found in the monument. Here we discuss the details of the calculation and its implications with some essential contextual background.

1 The decimal number system.

The ancient Babylonians used a numerical system based on the number 60, referred to today as the $sexagesimal$ system. But, theirs was not strict base-60 number system, as it was a hybrid of an underlying base-10 compounded with the number 6. Both scripture (2 SAMUEL 21:20; 1 CHRONICLES 20:6-8) and archeological finds indicate that there was a time when 6-fingered humans and 5-fingered humans mingled together. Even today, there are a small number of individuals around the globe that have 6 fully functioning fingers on each hand. Because the population of 6-fingered individuals is so small, relative to 5-fingered men, there's a tendency to forget that they even exist. Owing to the small population, there's a view that having 6-fingers is just an oddity, and not quite normal, as it goes against the grain of the most popular trend in the genetic pool of mankind. Today, it seems like some kind of recessive gene, that pops up here and there on occasion among the population of otherwise ordinary men; or, often thought of as some genetic defect, or deformity, as if something went wrong in the development process of the phenotype during gestation. But the records indicate that in old times 6-fingers was also associated with people of large stature, often referred to as giants, and this would naturally give them a physical advantage over the 5-fingered men. A population of 6 and

5 fingered individuals walking together, where the 6-fingered ones ruled for a time over the 5-fingers, would naturally result in the emergence of counting rules that accommodate both the workers and the rulers. The 5-fingers would feel comfortable with base 10, but the 6 fingers would insist that their anatomy be accommodated too, and if they were in power at one time, they could easily impose their requirement on the whole population. This could explain how the Babylonians came to use the sexagesimal system, as they probably inherited it from a time when having 6-fingers was a distinct advantage in the population, and the rulers set the standards.

count on 5-finger's man hands : $10 = 2 \cdot 5$
count on 6-finger's man hands : $12 = 2 \cdot 2 \cdot 3$
least common multiple (LCM) : $60 = 2 \cdot 2 \cdot 3 \cdot 5$

The main disadvantage of having 6-fingers, however, was that very accompanying large stature, as being a giant also means requiring more food to sustain and survive. So, it's natural that the smaller physique 5-fingered humans would multiply and dominate eventually, eliminating most of the 6-fingered giants.

By the time the ancient Egyptian civilization emerged later, all reference to the 6 was dropped, and at the building of the Great Pyramid base-10 was the dominant numerical method employed in counting and measurement. Apart from the King's Chamber, which is 60 cubits in perimeter, all numbers used in the design of this monument reflect mostly the 5-finger dominance, with very few vestigial remnants of the old sexagesimal system. This means that the Ancient Egyptian builders of the Pyramids had a similar concept to the modern idea of orders of magnitude based on powers of 10. The surviving records of hieroglyphic and hieratic text show special glyphs for 1, 10, 100, 1000, 10000, 100000, and 10^6, for example, while Egyptian fractions of the form $\frac{1}{n}$, where n is integer, were common in Egyptian arithmetic. Therefore, the Egyptian architects were certainly in possession of the conceptual numerical tools to help explain how the message, relating to what we would call Quantum Physics today, could be encoded in their monument; as we will rely on this particular decimal representation and it's corresponding idea of significant figures in part of the argument developed below.

2 Being open to a new idea.

Many observers of the Great Pyramid have noted that the design and measurements found there seem to implicate one theory or another as the intended plan. The critic of these observations usually points out that it is not possible for all these theories to coexist simultaneously, given the finite number of parameters available to fit; and further argues, by suggestion, that one can pick any object and find similar type relations of its measurements and design to many varied theories, but such observation does not imply any intent on the part of the architect to follow these ideas in design. We can refer to this objection colloquially as 'finding faces in the clouds.'

We say the critic argues by suggestion, since it is rare for the critic to prove his argument by selecting some random object and fitting various theories to it to demonstrate his point. Usually, the critic is content to point out the flaw in the arguments of others, as a pure thought experiment. But because of the bias inherent in the belief, there is unwillingness to extend the imagination to find and uncover the relations himself, even as an educational exercise. This is part of human nature. If we don't believe something is possible, we are unlikely to spend the time and energy to go look for it. And if we don't believe something is possible, we will employ our imagination to find counterarguments to debunk it whenever someone dares to suggest it.

To a certain extent the proponent is curve-fitting a set of data to some theory, and the critic opponent is suggesting one can fit many curves to many things, but that doesn't mean nature or designer had any knowledge of those curves or used them in the design plan.

But all knowledge proceeds by employing the imagination to introduce the new with some favored argument, and then counterargument to select the best among the new ideas, to build up that mental framework with which we interpret the world.

We need both proponent and critic. The proponent expands knowledge, and the critic shrinks it back down to what is essential, keeping the balance, so to speak. So, first we have to discard all we think we know about the past, to keep an open mind when we look, otherwise we cannot see what is right in front our eyes, and should be immediately obvious, since our belief tells us in advance that such things are impossible.

3 A principle of convergence out of chaos.

The first general idea that seems to be conveyed by the Great Pryamid, is that nature is neither this nor that, but that the things which emerge out of the background of chaos do so at points of convergence, where many discrete theories seem to apply just as well to describe the emergent phenomena. In a sense, chaos is conserved, as the forms that appear, out of the primordial chaos, maintain that multiplicity via uncertainty and ambiguity by establishing the forms at the crossing points of the possible distinct theoretical models that the mind could use to describe the apparent order found in the phenomena.

For example, in 1859 the English antiquarian John Taylor described two different theories that could explain the Great Pyramid. One theory suggests the designer based his construction on the PI ratio, so that the perimeter of the square base would have the same length as the circumference of that circle drawn with the pyramid height as radius. This was a long established problem in the sphere of mathematics: to find that rectilinear length measure equivalent to the curved length of an arc, in order to measure the circle. It was one of two variations of the classical problem of *squaring the circle*; the other variant being the problem of finding the square with same area as a given circle. Because of its long history, it was a very familiar idea, and thus not surprising why it would be one of the first things someone would consider when examining the monument. Apparently, Taylor didn't originate this observation, but he published it along with a second competing theory based on the PHI ratio in the same text.(1) Taylor noticed that the somewhat vague statements made by the Greek historian Herodotus could be interpreted to mean that the area of the triangle face on the pyramid was intended to be equal to the area of the square on the pyramid's height. This would immediately lead to the PHI ratio being found in the monument, and provide an alternative explanation to the PI model. Of course, these ratios, PI and PHI, are the usual *irrational transcendental* and *irrational algebraic* numbers with the familiar representations,

$$\text{PI} = \pi = 3.141\cdots \tag{1}$$

$$\text{PHI} = \phi = 1.618\cdots \tag{2}$$

Now, both of these theories cannot be true at the same time, if we think like classical scientists : that the measurements of the pyramid dimensions can only implicate one or the other. They give 'theoretically significant' different values for base angle of the pyramid, which is the angle the triangle face makes with the horizontal plane. If one is true the other must be false, and visa versa. This is just classical logic, based on the binary true-false paradigm.

However, we notice a peculiarity in the pyramid's construction. While the PI-model and PHI-model give different values for this base angle, the difference is small, and apparently less than the error or variance found in the actual construction of the monument, so that it is not possible to be certain which of the two models should be the true one. It appears that the architect of this monument introduced deliberate variances in the dimensions, so that one could not favor the PI-model over the PHI-model, suggesting that the intention was to implicate both models simultaneously, as if the Pyramid is a sort of hybrid between PI and PHI.

This hybrid idea is possible only because of a natural coincidence that finds PI and PHI meeting approximately at a sort of cross over point in the associated geometries connected with these ratios.

Further to this point, one notices that if one removes a capstone of about $1/16^{\text{th}}$ the size of the pyramid's dimensions, and requires that the volume of the truncated pyramid below be equal to a cube of that height, one again finds another model that could explain the base angle, coming close once again to the other two. This CUBE-model is yet a 3rd geometry that could explain the pyramid, and has the added benefit of explaining why the architect might have left off the capstone.(2)

So, we have at least three theories : the PI-model, the PHI-model, and the CUBE-model, all competing for the spot as best fit to the construction, because the geometries of these three all coincidentally meet approximately at the same base angle, and are all within the apparent error of construction.

Depending on who looks and examines the Great Pyramid, a different model is imagined to be fit. One person thinks it's a PI-model. Another thinks it's a PHI-model. A third thinks it's the CUBE-model. The observer finds the model he or she is predisposed to find. This reminds us of Quantum Physics, where one experimenter would obtain results when measuring that differs from another experimenter, or same experimenter at a different time, and the conclusion we have formed after seeing how this all works, is that nature is a sort of hybrid, which we explain using a wavefunction, and the observer finds one discrete state or another discrete state when he looks, but the true underlying nature of things is really a hybrid of all these observations.

So, is the Pyramid intentionally the 'model in stone' of the idea of a Quantum Waveform from which discrete distinct geometric observations can be inferred? Or is it just a fluke that it appears to be like that today, now that we understand the world a bit better with our modern ideas?

If one was to accept the challenge today, to design and construct a monument in stone, to preserve the fundamental discoveries of quantum physics, how would one do this? The Great Pyramid suggests a way, and introduces us to the idea that perhaps someone did this very thing a long time ago, and left the monument there for future generations to find and interpret correctly when they were ready to see the message.

But it's not just the geometry of the monument that seems to admit various discrete theoretical models, being at the junction or cross-roads of several geometries, since the location of the Great Pyramid is also at a junction, or cross-roads at Giza.

West of the pyramid we find the largest Sandy deserts, an area where all the particles of matter that make up the Earth's surface are broken apart into small discrete fragments, while East of the pyramid we find mostly Rocky desert, where all matter on the surface is generally fused together into continuous chunks of solid rock. So the pyramid is at the boundary of these two distinct geophysical features of the land, sitting there at the crossing point.

Again, North of the pyramid is the fertile alluvial delta plane, the so called "black lands" of Egypt, where crops can be readily grown; while South of the pyramid is the hot and dry arid barren "red lands" of Egypt, where growing crops is almost impossible except right along the banks of the Nile. So again, the Great Pyramid sits at the boundary between North and South on a critical essential fertility dividing line between life and death. Therefore, the junction, or meeting place, or crossing of features, is in the choice of geographic location for the pyramid, in addition to the specific geometry selected for the shape. It's therefore hard not to come to the conclusion that by repeating a theme the architect intended to convey that specific idea, and draw our attention to it.

Standing on the East bank of the Nile, looking West, one sees the Great Pyramid looming large, as an apparently definite geometric form appearing to rise up out of the vast featureless sandy desert, like the emergence of a distinct notable phenomenon out of the primordial chaos from an indistinct formless background. It's hard not to see the correspondence of the idea. But, upon attempting to examine that geometric form more closely, applying our measuring tools to pick apart the mystery of the form, its exact pattern eludes us, for it then seems to be neither this nor that, depending on how we look. This is exactly what happens when we examine almost anything in nature, and attempt to get our knowledge of it down to a higher level of precision than the initial simple casual arm's length observation. Standing West of the Pyramid, looking East, we again get a similar impression, only now the rocky landscape forms the indistinct background; the contrast between East and West being the 'continuous' verses the 'discrete' themes of nature, provided by 'rock' and 'sand', and the Pyramid sits at the junction of that discrete vs continuous boundary of physical reality.

Standing South of the monument, looking North, one sees the Great Pyramid rising up out of the fertile soils of the delta plane, the *plane of life*, as if some kind of living thing grew up right there at the very spot that marked the apex of the delta, from the beginning of time. While, standing North of the monument, looking South, one sees the giant monument sitting absolutely motionless, as if death overcame that growing thing, and it became stuck there standing at the start of the arid plane, the *plane of death*, looking like a giant fossil of some kind of dinosaur of an earlier time, all encased in limestone deposit, as if it had emerged a long time ago from its grave in a watery world of an even more remote past, now silently marking the beginning of the red lands.

Hence, it's hard again, not to see that intended message, that the pyramid sits at the cross-roads of life and death. A Tibetan Buddhist might call this the bardo, or the intermediate state between life and death; a state that is neither life as we know it, nor actual death as one would imagine extinction to be. But, a modern scientist would declare a person is either alive or dead, using his classical logic, and modern science does not admit any intermediate state or hybrid of the two; not yet, anyway.

This is not to say that the western mind isn't familiar with ideas of such types of hybrid; for in the mythology of the west we find the idea of the vampire, a creature that is both dead and alive, walking among the living and walking among the dead simultaneously; or again, the idea of the zombie, a creature that is actually supposed to be dead, but moves about as if it still has life in it, with just the basics of mind, lusting after human flesh, and so on. These mythological creatures are just exaggerated archetypes representing extreme limits of what we generally think of human consciousness today, where man himself is seen as the hybrid, with some people more alive than others, and some seeming more dead. But, like the cat looking at itself in the mirror and thinking it's seeing another cat, man has exhibited different levels of awareness and consciousness distributed quite unevenly among the human population. This has led to the emergence of various religions. The Christians claim that until a person is born again, by receiving the holy spirit within themselves, they are actually walking about dead, and are unaware of it–dragging the lifeless body of sin around as they walk, as if they had a sack of potatoes on their backs; a burden that can only be lifted by willingly accepting Jesus into their life, so that he may wash away their sin. The Gnostics claim that until one discovers oneself, by temporarily shedding the physical body–which is a garment of forgetfulness that blocks our memory of who we really are–and unleash the hidden spiritual body that each person has being unaware, one has not experienced true life at all; While the Tibetan Buddhist calls this initiation into the between, where the lama's primary role is then seen as "introducing one to the reality," a theme repeatedly found again and again in the *Tibetan Book of the Dead*, and so on. So, the ideas are there, all wrapped up in parables and metaphors found in myth and religion, but not accepted into the domain of science, as of yet, because all science requires one to be able to make '*measurements*' to test the validity of any given thesis or point of view, that the scientist would be expected to take seriously. If you can't measure it, it's just good for entertainment, philosophy, or religion, but not science.

So, apart from making interesting observations of a conceptual nature about the location and shape of the pyramid, if indeed there are intentional references to ideas found in modern quantum physics there, we must show how this can be demonstrated through measurements. That leads us to think about what sorts of measurements we can make concerning the monument, that might reveal some truth we could identify with things we know today.

4 A minimum energy of construction.

Almost every visitor to the Giza plateau marvels at the giant monuments, and ponders how on Earth man with just primitive tools was able to build these things. About the first question that occurs is how much time and effort it must have taken to complete this construction. The historian Herodotus pondered this same thing, and made enqiries of the Egyptian Priests who he interrogated in 500 B.C.(3) The answer was 100,000 men using just manual labor, with the help of some primitive wooden lifting machines, took 30 years from start to finish. The first 10 years was spent just preparing the platform at the Giza site and building the road and ramps leading up to the platform, in order to transport the stones to the site, and the pyramid itself was then erected in the remaining 20 years. Of course, at the time the Priests were telling Herodotus this story, the pyramids had been around for at least 2000 years already, and it's unclear how seriously one should take the claim that they actually knew how the construction was performed and didn't just make it all up on the spot to seem knowledgeable and wise to their foreign visitor. Moreover, we know from psychological studies today, that when any story is told and re-told over time, passing from one individual to another by word of mouth, by the mere "retelling" it undergoes transformations that distort and changes it, so that it becomes unrecognizable as representing the facts in the initial content after just a short time. There are no written records, before Herodotus, that give those details about the construction. So, this information came down by word of mouth to the Egyptian Priests, and then to Herodotus, who then wrote down what they said. Nevertheless, Herodotus is what we have as a starting point.

At rest, just doing nothing, the average human is said to burn about $100\ Watts$, which is 100 Joules of Energy per second. To perform physical activity, the body must burn more than this, so that a top athlete sprinting could burn as high as $2000\ Watts$, but only maintain this effort for a few minutes. The body is typically capable of sustaining about $350\ Watts$ over a longer period of time. So, one can say that $350\ W - 100\ W = 250\ Watts$ is available as periodic sustained energy per unit time, taking rests now and then, for the performance of work. Of that $250\ W$, part is just lost to extra body heat and chemical processes in the extra exertion over the rest state, and the remaining part goes into the work being performed, mechanically transferred by the action of the muscles overcoming resistive forces.

Let's say the man is 80% efficient at this point, so that $.8 \times 250\ W = 200\ W$ will be converted mechanically into whatever project he is doing, and becomes part of the energy of the project itself. If one man works a total of 8 of the 24 hours per day, with the required rests to recover from physical exertion as required, then in one day one man can contribute $8\ hr\ \times\ 60\ \frac{min}{hr}\ \times\ 60\ \frac{s}{min}\ \times\ 200\ W = 5.76\ \times\ 10^6\ J$; so, $100,000$ men in 20 years contribute: $100,000\ \times\ 20\ \times\ 365\ \times\ 5.76\ \times\ 10^6\ J = 4.2\ \times\ 10^{15}\ J$, to build the Great Pyramid itself, and just add half that again for the total 30 years project of preparation plus pyramid. This gives us an estimate of the maximum amount of energy available from the human labor pool to perform all the construction work, using Herodotus' report.

Now, when considering all construction work, there are many unknowns, and one has to make assumptions here and there about how the work was done. For example, the stones were believed to be transported on boats on the river Nile. But did the workmen paddle, or use sails and wind power to propel the boats from quarry site to Giza? If they used wind power, that energy is not required from the human labor pool, because the men are using renewable wind energy, which ultimately comes from the sun, and so they need less human labor to perform all the work. Also, while there is the dominant theory that men cut all the stones at the quarry site and transported the large blocks to Giza, there is at least one competing modern theory that suggests, that in some cases, the men just crushed the rock at the quarry site into powder, transported the powdered material, and manufactured the blocks on site at Giza from a type of geopolymer cement, much like we do in construction today. The considerations of quarrying, transportation, block formation, etc.. include more variables that produce uncertainty than we really need to consider in the current discussion, as there is at least one part of the construction project that can be attributed a rather definite energy, which is what we are mostly concerned about.

We think about what "message" the Great Pyramid is intended to convey, after it's completion; That is, the intention of the architect, as concerning the finished monument. A visitor to the Giza site, observing and contemplating the Great Pyramid, only sees the finished product, and can't be expected to know anything about it's construction, such as quarrying, transportation, lifting, block formation, and so on. What the visitor sees, is apparently a 5-sided geometric pyramid, that is truncated at the top, seeming to be all built from bright white shiny tura limestone, the casing stone that once covered all of the surface of the pyramid when it was initially completed.

Now, we know that the tura limestone was only on the surface, and underneath that, the so called bones of the pyramid, which are exposed today, is made up of a different type of yellowish less pure limestone. We also know that parts of the interior of the pyramid uses granite, such as in the King's Chamber; and that the pyramid is not all built from stones, but was built over a small mound or hill, rising up in the center of the pyramid, which can be seen when descending the Well Shaft, the worker's escape passage that connects the descending passage to the bottom of the Grand Gallery. This shows that at least up to about the 7th step layer internally the material is just the natural rise formation of the land. The builders didn't have to put stones there. That work was done by nature already a long time ago. However, all this aside, the visitor to the Giza site is induced to think of the pyramid as sitting on a level flat platform, all made of white tura limestone, in the geometric shape of a truncated pyramid. So, what we are to think about, is that image that is conveyed by what we are supposed to see when we look, and discard all the particular extra details that we know about the internals, which are only known because the pyramid is in a state of decay today.

Whatever we might think about the quarrying, transportation, and block formation work, there is a definite amount of energy required to raise the stones from the Giza plateau in the Earth's gravitational field, to the heights they are placed to form the pyramid structure. If we assume all the stones are made from the same material, of known density, then the mass of such a pyramid can be computed, and this energy can be computed with less uncertainty than all other parts of the construction project. This then is the minimum energy required to erect the Great Pyramid, since every other contributing activity can only add to the total energy required. Now, we should emphasize, this minimum energy we are talking about, is that energy a visitor to the site would contemplate after witnessing the monument, observing only its external features. The visitor doesn't know that some granite stones were raised, only tura limestones. The visitor doesn't know that some internal parts of the pyramid are from a natural rise in the land formation, around which the pyramid was built. The impression the visitor gets is that all the pyramid is made of tura limestone, and was built up from a horizontal flat plane, erected up into the air, to the truncated summit. That gives the visitor a definite geometric shape and one material density, that of tura limestone, to factor into his calculations.

Now, limestone is a material that is found in nature with variable density. It is mostly Calcium Carbonate, CaCO_3. But depending on the quarry site, the limestone may be porous, containing pockets of air, reducing the density, or may contain other minerals, chemical compounds, and materials such as fossils and bones, that can raise the density. The architect obviously knew this, and by picking the purest white limestone for the casing stones, the suggestion being conveyed by choice of material is that the visitor contemplate the purest form possible to get that definite unambiguous density to include in his contemplative calculations, which therefore would be pure Calcium Carbonate itself; and the most stable form of CaCO_3 has a definite density of $2.71\ g/cm^3$.

Now we have a reasonably definite shape, and reasonably definite density of material, implicated by contemplating what can be seen, so we can calculate the architect's intended theoretically implied minimum energy required to raise all the stones from the platform to construct the pyramid.

Physicists have a neat trick to compute the energy required to raise all the stones in a structure like the great pyramid. Applying the principle of linear supposition, all one needs to do is find the mass, M, of the whole pyramid, and the height of the center of mass, H_{cm}, of that geometric shape, presuming uniform density, and simply compute the product with the acceleration due to gravity : $E = M \cdot g \cdot H_{\text{cm}}$. The height, H_{cm}, of the center of mass of any pyramid like the Great Pyramid is always $\frac{1}{4}$ the height to apex, $H_{\text{cm}} = \frac{1}{4} \cdot H$, regardless of the slope the face makes with the horizontal plane. The volume of such a pyramid is $1/3^{\text{rd}}$ the area of its base times the perpendicular height; so if we conveniently describe the base as having side length of "$2L$", then volume : $V = \frac{1}{3} \cdot (2L)^2 \cdot H$. For a density of material, ρ, the mass is, $M = \rho \cdot V = \rho \cdot \frac{1}{3} \cdot (2L)^2 \cdot H$. The minimum energy to raise the stones is then,

$$E = \rho \cdot \frac{1}{3} \cdot (2L)^2 \cdot H \cdot g \cdot \frac{1}{4} \cdot H \tag{3}$$

But, this is for a 5-sided pyramid, and the Great Pyramid is intentionally missing the capstone. It has six sides, since the top is flat. So, we need to subtract that energy that would be required to put the capstone there that's included automatically in this calculation. The capstone is a similar geometric figure to the complete Pyramid, and all it's linear dimensions are $1/16^{\text{th}}$ of the Great Pyramid's blueprint design, and so it too has a center of mass $\frac{1}{4}$ of its height above its base. But since its base is on top of the truncated Pyramid, we have to add that height, $H' = \frac{15}{16} \cdot H$, to get its actual height above the ground; so the energy, E', to build the truncated pyramid is,

$$\text{Pyramid Blueprint} \qquad - \qquad \text{Capstone}$$

$$\begin{aligned} E' &= \rho \cdot \frac{1}{3} \cdot (2L)^2 \cdot H \cdot g \cdot \frac{1}{4} \cdot H \quad - \quad \rho \cdot \frac{1}{3} \cdot (\frac{1}{16} \cdot 2L)^2 \cdot (\frac{1}{16} \cdot H) \cdot g \cdot (\frac{1}{4} \cdot \frac{1}{16} \cdot H + \frac{15}{16} \cdot H) \\ &= \frac{1}{3} \cdot \rho \cdot H^2 \cdot g \cdot (2L)^2 \cdot \frac{1}{4} \cdot (1 - \frac{61}{16^4}) \end{aligned} \tag{4}$$

5 Comparing Pyramid Models to Data.

Now in the RMP (The Rhind Mathematical Papyrus, circa 1650 B.C.)–which illustrates how the ancient Egyptians actually thought about some of their calculations–there are problems on the Pyramid (Problems 56-60) where the slope is measured by a parameter called the '*seked*,' and the *seked* of a Pyramid is given there as so many palms plus fingers or digits$^{(4)}$. It was usual when describing the slope to note the horizontal displacement per cubit of height, where the cubit was understood to be the royal cubit of 7 palms, and each palm consists of 4 digits. A seked of 5 palms plus 1 digit, for example, would represent a slope ratio of, $s = L/H = 5.25/7$. This allows the Egyptian architect to construct several slopes for his pyramid, but with the restriction that such slope will fall near to one of the several discrete values constrained by the smallest unit of the digit. Egyptologists have noticed that many pyramids in Egypt actually have slopes that could be explained by using the seked system described in the RMP, as the slopes follow closely one of the discrete values determined by picking the seked consisting of an integer number of palms and digits. The Great Pyramid's slope, for example, could be explained by a seked of 5 palms 2 digits, or $5\ 1/2$ palms, as the nearest fit to the actual slope, corresponding to a slope ratio of, $s = L/H = 5.5/7$. So, we have yet another model that can describe how the architect went about his plan, and we'll call this one the RMP-model, since that's where the evidence of such calculation shows up in the surviving ancient records. This RMP-model value of, $s = 5.5/7$, can actually be thought of as just a variation of the PI-model with the approximation, $\pi \approx 22/7$, instead of the true high precision value of PI. But, in general, for any such slope parameter, s, we have,

$$s = L/H \quad .: \quad L = s \cdot H$$

$$s \;=\; L/H \;=\; 5.5/7 \;=\; 11/14 \;=\; 22/28 \;=\; 1/4 \cdot 22/7 \;\approx\; \pi/4$$

Theoretical data from models:

PI-model	$s = \pi/4,$	$\psi = 51^\circ 51' 14''$	
EULER-Model	$s = \cot(\frac{360}{4} \cdot (1 - 1/(1 + e/2)))$	$\psi = 51^\circ 51' 02''$	$\cdots \quad e = 2.718281 \cdots$
ARRIS-model	$s = (10/9) \cdot (1/\sqrt{2}),$	$\psi = 51^\circ 50' 39''$	
RMP-model	$s = 5.5/7,$	$\psi = 51^\circ 50' 34''$	
PHI-model	$s = 1/\sqrt{\phi}$	$\psi = 51^\circ 49' 38''$	$\cdots \quad \phi = (1 + \sqrt{5})/2$
CUBE-model	$s = \sqrt{(\frac{3}{4} \cdot 15^3/(16^3 - 1))}$	$\psi = 51^\circ 49' 30''$	

$$\text{Range of angles} \quad : \quad 51^\circ 51' 14'' - 51^\circ 49' 30'' \quad = \quad 1' 44'' \quad < \quad 2'$$

Euler too? Now, somewhere it was first noticed that when taking the ratio of the base angle to corresponding apex angle, and doubling the result, Euler's number $e = 2.718..$ was approximately obtained. For example, if we take the PI-model, $s = \pi/4$, and compute the base angle, $\psi = \arctan(1/s) = 51.853974..$, find the corresponding apex angle, $90 - 51.853974 = 38.146026$, and double their ratio, $2 \times 51.853974/38.146026 = 2.718709... \approx e$; a difference of 0.0004 or 4 parts in $10,000$. This is a purely numeric coincidence without any apparent parallel geometric explanation attending to it that would satisfy the mind that the model conveys some kind of useful information, unlike the cases with the PI, PHI, and CUBE models. However, we've included this, as it was a valid observation sample of something factual that produces a base angle in the range being considered, if we adopt Euler's number "e" as the guiding design parameter making it exact rather than approximate. Measuring angles in 360 degrees, the usual equivalent slope parameter can then be written, $s = \cot(\frac{360}{4} \cdot (1 - 1/(1 + e/2))$, where "$360$" could be replaced by "$2\pi$" if the angle measures were being given in radians instead of degrees.

Is Arris the Key? Starting in about 1867, several people began writing on the observation that the arris, i.e. the line at the edge of the pyramid going up from base corner to apex, was in the ratio $10/9$ or $9/10$, depending on which way the ratio was given: 10 horizontal to 9 vertical. In his book "*The Shape of the Great Pyramid*," author Roger Herz-Fischler devotes a whole chapter (Chapter 14) discussing the various proponents of the arris 9/10 theory.(5) It appears that William Petrie, the father of W. Flinders Petrie, may have been the first person on record to make this observation, after reading Piazzi Smyth's 1st edition of "*Our Inheritance..*"(6). W. Flinders was therefore well acquainted with this idea. One thing we might add here to that conversation is the observation made by the surveyors that the diagonals of the socket square of the Great Pyramid meet at almost an exact right angle, even though the sides of this square aren't really square at all. This has led to the speculation that the diagonal line from S.E to N.W and diagonal line from N.E to S.W were probably the first things established at the Giza plateau in order to orient the pyramid. Since the arris is in the same vertical plane of these base diagonals, it is possible that there was some special focus on the arris during construction. As has been pointed out, the arris is the only geometric feature on the pyramid that is actually a visible physical straight line running from the base to apex. This makes it convenient to observe when the construction engineers are deviating from the plan. The idea then, is that the diagonals are first set, the corner sockets are established on these diagonals, and then construction begins from the four corners going up to keep things moving in a straight line, while the sides are filled in with stones after the corners have been established on each level as the work progresses. This makes it more expedient to have a simple ratio like $10/9$ to follow the arris up during construction. So, from a purely practical consideration, despite all the theories that might be intended to be implicated by the architect, when it comes to erecting the structure, the construction engineers probably used some rule of thumb rational approximation for the arris edge to build the monument. At any rate, we've included this idea as the "ARRIS-model" in this list. There doesn't seem to be any insightful geometric message being conveyed by a rational number used for this edge, as it appears to be a purely utilitarian factor in construction. Since the arris runs in the plane of the diagonals, we have to adjust its slope parameter by a factor of $\sqrt{2}$ to get the corresponding slope the triangle face makes with the base. For the arris of 10 horizontal to 9 vertical, our slope parameter of the triangle face is then, $s = (10/\sqrt{2})/9$.

Measured data from surveys:

			Weight
Petrie's measurement of Casing stones, in situ, N. side, by theodolite:	$51^\circ 46' 45''$	$\pm 2' 7''$	7
(To three points on top and three on base.)			
" " " by goniometer and level	$51^\circ 49'$	$\pm 1'$	2
" " " by steel square and level	$51^\circ 44' 11''$	$\pm 23''$	0
" " 5 overthrown by goniometer	$51^\circ 52'$	$\pm 2'$	0
" " 18 fragments, all sides, goniometer	$51^\circ 53'$	$\pm 4'$	0
(All above 2 inches in shortest length.)			
N. face, by entrance passage mouth	$51^\circ 53' 20''$	$\pm 1'$	10
N. face, by air channel mouth	$51^\circ 51' 30''$	$\pm 20''$	5
N. face, weighted mean	$51^\circ 50' 40''$	$\pm 1' 5''$	
S. face, by air channel mouth	$51^\circ 57' 30''$	$\pm 20''$	

(Table of survey data from Ch.vi, sec.24, pg.42, Petrie 1883.(7))

Petrie makes many measurements of the slope of the pyramid while doing his survey work at Giza; and after he collects his results, he sits down and examines the data and decides he trusts some of his own data points more than other measurements he made, and he assigns a confidence weight of '0' to data he decides are uncertain, to be cast out, and assigns a positive weight number like 7,2,10,5, to give a sense of how much he feels he trusts those data points in his records. But, he nevertheless publishes all the data in this table, whether he thought the data were acceptable or not. Then he computes the weighted mean of the data and reports such mean for the North face, and gives one sample measure on the South face.

Then Petrie renders the following personal opinion on the data,

> "*On the whole, we probably cannot do better than take* $51^\circ 52' \pm 2'$ *as the nearest approximation to the mean angle of the Pyramid, allowing some weight to the South side.*" – Petrie 1883, #24-25.Ch.6

Here it seems Petrie takes $4/5$ of the N.Face mean $51^\circ 50' 40''$ with $1/5$ of the S.face sample $51^\circ 57' 30''$, which would give him an average of the North and South faces together of $51^\circ 52' 02''$, then he discards the extra $02''$, tosses this into the error instead, and reports the $51^\circ 52'$ with a guess of $\pm 2'$ which he feels should be a reasonable estimate of what is "achievable" given his widest measurement error was $\pm 2' 7''$ among the data points which he accepted as good.

What is interesting, is that Petrie is well aware of the RMP calculation on the Pyramid from the Rhind papyrus, as this text was discovered in about 1858, with many scholars writing articles on it, and a complete translation of all its 84 problems had already been published in Germany by 1873, some 10 years before Petrie publishes the results of his survey expedition to Giza in 1883.

Petrie N. face mean	$51^\circ 50'40'' \pm 1'5''$
ARRIS-model; $s = 10/9 \cdot 1/\sqrt{2}$	$51^\circ 50'39''$
RMP-model; seked based calc.	$51^\circ 50'34''$

So it is notable that his choice of weights just so happens to produce an average value for the North Face of $51^\circ 50'40''$, which is remarkably close to the very value of $51^\circ 50'34''$ that the RMP-model would produce; and even closer to the ARRIS-model, first proposed by Petrie's own father back in 1867.

It's hard not to think that he was influenced by his 'expectation' of what the slope should be, from this previous knowledge; and his level of comfort with his own data, as illustrated by his choice of weights, was really guided by an attempt by him to fit the data to this expectation, once he sat down and started his analysis when going over his notebooks. Petrie had a definite preference for a model in mind, and that influenced his choices of data points. This view seems to be confirmed, by his own writings, where he says,

> *"For the whole form the π proportion (height is the radius of a circle = circumference of Pyramid) has been very generally accepted of late years, and is a relation strongly confirmed by the presence of the numbers 7 and 22 in the number of cubits in height and base respectively; 7:22 being one of the best known approximations to π."*(*)

* – pp.92-93, Chapter IX, A Sketch of the History and Design of the Great Pyramid, in "THE PYRAMIDS AND TEMPLES OF GIZEH" of 1883, by W. M. FLINDERS PETRIE, New and Revised Edition, 1990; with an update by Dr. Zahi Hawass.(8)

Notice that John Taylor had published both PI and PHI models in 1859, and everyone in the antiquarian field knew of these ideas by 1883. But, there was no evidence that the ancient Egyptians knew anything about the PHI ratio from what survived of their writings, and certainly didn't appear to know PI to anything better than the approximate value of $22/7$. So whatever measurements Petrie made at Giza, had to be used to form conclusions that were consistent within that framework of what Egyptians were believed to know by the academia of the time. The RMP was there, and provided definite evidence for the Egyptians thinking the *seked* way, and thus, $s = 5.5/7$, could not be denied by anybody in the field, and this value was consistent with the very approximate value, $\pi \approx 22/7$, that many Egyptologists were willing to grant the ancient Egyptians knowledge of, so it was a safe bet to make sure the final results reported agreed with this generally accepted framework. The power of that acceptance caused Petrie to doubt and cast out some of his own measurements. The official explanation that Petrie gives, for why he was discarding some of them, is that he was not sure whether the flat horizontal bottoms of these casing stones that he found laying about were actually placed in a true horizontal plane when they were "in situ" on the core's blocks, since he noticed that some of the top surfaces of the exposed core blocks were not exactly horizontal, but inclined a bit by something like $\frac{1}{2}^{\circ}$ to 1° of arc, being higher towards the face and dipping inward towards the back at the core frame. Because he could not prove that they were horizontal when in place, he leaves them out of his calculation for the mean. With this argument, together with his choice weights for the remaining data he accepts, he computes a mean suspiciously close to the RMP-model. Petrie makes no mention of the PHI-model, but alludes to there being other "dubious" theories that he deliberately left out of his work; for while being "true," in his opinion, they lacked value because they didn't represent themes that were repeatedly found in other parts of the structure. In essence, they were just actual accidental coincidences.

In otherwords, Petrie is not simply measuring what is there. He is measuring with anticipated expectation of what the results should be, already in mind; and this is interfering with his objectivity. So, when he has to make a subjective judgment, because of some uncertainty, he leans towards the conclusions that would fit his preferential expectation and be in harmony with the 'group think' of his peers.

That this preconceived model expectation is part of his general frame of mind is also revealed in his criticism levied on the 1925 Egyptian Survey of Cole. Petrie measures the base of the Great Pyramid with the idea in mind that it should be a square. So his measurement setup, when surveying, attempted to fit the best square to the monument's base. Cole measures each side independently, requiring the introduction of many more survey points to triangulate the lengths, and Petrie's criticism was that adding more survey points just increases the uncertainty in the results, and reduces the confidence in the computed values; while when using fewer survey points like Petrie did, would increase the confidence in the computed results. Each time you add an extra unique survey point, you add an additional uncertainty from the measurement at that location, so when you compute a result using many such survey points, the size of the uncertainty grows wider in the computed result. On this basis, Petrie defends his own 1883 survey, and claims he has more confidence the 1883 results than the newer 1925 results of Cole.(9) But, of course, Petrie can only use fewer survey points, if he has a simpler model in mind, like the pyramid should be a 'square base' , and is just measuring to fit that model. For example, it is known that the core of the pyramid actually has 8 sides, as each side is indented at the middle; although it is believed that when the casing stones were present this feature wasn't there on the finished monument, because the few casing stones that were found "in situ" at the base of the Pyramid did not show any such indentation even though these few stones were exactly in the middle of the face. But, if you believe the number of sides should be 8, instead of 4, you need to add more unique survey points to capture that variation in shape. So, the number of survey points is partly dependent on the assumptions you make about what exactly it is you're measuring. For Petrie, it's a square. For Cole, it's just a closed plane figure with 4 straight lines without any presumption about the angle these lines are supposed to intersect; in other words, just an arbitrary quadralinear polygon; as Cole writes in his Survey of Egypt,

> ' *These differences in azimuth are due to the fact that the new azimuths are found from the actual directions of the sides determined from the excavated pavement, whereas those of Prof. Petrie are of a hypothetical base obtained by computing "a square that shall pass through the points of the casing found on each side, and having also its corners lying on the diagonals of the sockets."*'(10)

Despite thinking that the base of the Great Pyramid is a square, and then remarking in his notes on how perfectly the square it is–attesting to the observation that it was likely even more square originally than he was able to measure it–Petrie goes on to note in his report that the "*top is, rather strangely, not square, although it is so near to the original apex,*" and writes that "*we have no guarantee that the Pyramid was equiangular, and vertical in its axis.*" In his mind, therefore, the Pyramid goes from a nearly perfect square base to a non-square flat top, and so the angles of the four faces must by necessity be divergent one from another on the triangular faces, to accommodate that observation, and by this reasoning, "*It is clear that the South face should not be included with the North, in taking the mean.*" (Ch6, sec.24-25, pg.43, Petrie 1883) . This suggests, of course, that each face may have it's own design angle different from the other faces. So, while Petrie does give an estimate of what he thinks the overall mean of the base angle "could" be, combining the North with South, he really only has confidence in his North face measurements, which is the one that comes suspiciously close to the RMP and ARRIS models.

But, as we can see, regardless of how Petrie felt about his measured data afterwords, all the actual survey data he recorded at the time of the work in 1880-1883 at least falls in the absolute range,

$$51^\circ 43' 48'' < \psi \ \text{(base angle)} < 51^\circ 57' 50'' \qquad \text{... a spread of } 14' 2'' \text{ of arc;} \tag{5}$$

a range wide enough to include all the models we've listed above, so that The Great Pyramid appears to have sufficient built in variances in the actual construction of the structure to accommodate several theoretical models.

The Great Pyramid then stands as a clever mathematical puzzle, with several resolutions available for the viewer to unpack. As the viewer finds one model, and studies its implications, information about the universe is revealed. When he considers another model that seems to also fit, he finds out more things about the world. The Pyramid is acting as a silent classroom instructor, directing the mind of its students to consider this and that, which, once the mind travels down that path, new revelations are uncovered. This is exactly what a teacher does. There are infinitely many things a student can apply his mind to learn, and the role of the teacher is to direct that inquisitive mind to the most useful things he should be spending his time to learn first. From the shear size of the monument, the Pyramid grabs attention, as if calling out to the passersby, "Come, look here! I have things to reveal to you. All you have to do is think."

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6 Computing the Energy with known values of parameters.

Re-writing the energy expression in terms of the slope, s, and, Height, H, and rearranging, we have,

$$\begin{aligned} E' &= \frac{1}{3} \cdot \rho \cdot H^2 \cdot g \cdot (2 \cdot s \cdot H)^2 \cdot \frac{1}{4} \cdot (1 - 61/16^4) \\ &= (1 - 61/16^4) \cdot \frac{1}{3} \cdot g \cdot \rho \cdot s^2 \cdot H^4 \qquad (6) \end{aligned}$$

The first factor, $(1 - 61/16^4) = 0.999069\cdots$, is the adjustment for the missing capstone, which should otherwise be just the unit, 1, if the pyramid was constructed all the way to its apex. This has negligible effect on the calculation, but we leave it in there just for completeness, because it's part of our 'model under study.' Then, we plug in the values for all these parameters,

$s = 5.5/7$	RMP 1650 B.C., seked method, Problems 56-60
$H = 5776$ inches	Petrie. 1883 survey report, CASING OF THE PYRAMID, ch vi, Sect. 25, pg.43
$g = 9.8 m/s^2$	standard text book typical value
$\rho = 2.710 g/cm^3$	Handbook of Chemistry and Physics 95th Ed.(11) p.4-54, #559, ($CaCO_3$:Calcite; trigonal)
$= 2.930 g/cm^3$	" p.4-54, #558, ($CaCO_3$:Aragonite; orthorhombic)
$= 2.653 g/cm^3$	" p.4-54, #560, ($CaCO_3$:Vaterite; hexagonal)

Aragonite and Vaterite are metastable phases of $CaCO_3$ that eventually convert to Calcite which is the most stable form.

. :

$$\begin{aligned} E' &= (1 - 61/16^4) \cdot \frac{1}{3} \cdot 9.8 \,\frac{m}{s^2} \cdot (2.71 \,\frac{g}{cm^3} \cdot \frac{1}{1000} \,\frac{kg}{g} \cdot 100^3 \frac{cm^3}{m^3}) \cdot (\frac{5.5}{7})^2 \cdot (5776\, in \cdot 2.54 \,\frac{cm}{in} \cdot \frac{1}{100} \,\frac{m}{cm})^4 \\ &= (1 - 61/16^4) \cdot 2.5319 \times 10^{12}\ J \qquad (7) \\ &= 2.5295 \times 10^{12}\ J \qquad (8) \end{aligned}$$

As we can see, the energy required to raise the stones in this theoretical calculation is $2.5 \times 10^{12}\ J$, while the energy in the human labor pool over the 20 year period above was found to be $4.2 \times 10^{15}\ J$. That's about 1680 times more energy available than required to do the sort of work involved to just raise the stones. Even if we made adjustments for the real stone work, accounting for the difference from our fictitious pure $CaCO_3$ stone monument, we wouldn't use up 1680 times more energy to raise up the pyramid. We must remember though, that we still have to account for the quarrying, transporting, and block forming, platform leveling, road building, etc.., which are not part of our calculation, and which will also use up substantial amounts of energy; and a critic might argue that we were quite generous in calculating how much energy one person could contribute per day, and/or point out the belief that some expressed that the men only worked during the summer months during those 20 years, since this was their off-time when the flood made it impossible to do other things etc.. But, even so, we wouldn't expect those parts to even consume all the remainder of the 1680 times energy, or $1/4 \times 1680 = 420$ times (summer only work), that just raising the stones left back for such work. It therefore appears that the numbers that Herodotus reports are not unreasonable, as they are consistent at least with the energy requirement of the work (achieving the exhibited precision level in construction is another matter altogether). Given that many pyramids in Egypt were built by human labor, although of lesser quality than the Great Pyramid, the Egyptians would have a pretty good idea of how much time and effort it would take in terms of manpower to perform this type of work for something on that scale, and could easily have computed themselves the number of men and number of years required to erect the monument, even if they didn't in fact build this particular one themselves. They had sufficient exposure to making 'copies' of the pyramid form, that they could estimate the work fairly well. To date, well over 100 pyramids have been discovered in Egypt, illustrating the point that the ancient Egyptians were quite busy repeatedly practicing the construction of this type of monument.

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7 Planck Length in the Average Energy.

Having found the energy required to raise up the Great Pyramid, E', the next obvious thing we'd naturally contemplate to look at is the average energy per step, E_s, given that there are 203 steps in the construction. One way to think about this value is to compare it to that of a photon with same energy, and examine the corresponding frequency and wavelength.

$$E_s = E'/203 = 1.246 \times 10^{10}\ J \tag{9}$$

$$E_s = hf = hc/\lambda \tag{10}$$

now with,

$$h = 6.62607015 \times 10^{-34}\ J.s \tag{11}$$

$$c = 299,792,458\ m/s \tag{12}$$

we find,

$$\lambda = \frac{hc}{E_s} = \frac{6.62607015 \times 10^{-34}\ J.s \times 299,792,458\ \frac{m}{s}}{1.246 \times 10^{10}\ J} = 1.594 \times 10^{-35}\ m \approx 1.6 \times 10^{-35}\ m \tag{13}$$

$$\text{But, Planck Length*} = 1.616229(38) \times 10^{-35}\ m \tag{14}$$

* Particle Physics Booklet 2018$^{(12)}$ pg.8, Table 2.1: Astrophysical Constants and Parameters (http://pdg.lbl.gov)

To 2 significant figures, 1.6, and the exact order of magnitude, $10^{-35}\ m$, we find the average energy per step is none other than that of a Photon whose wavelength is the Planck Length ! Hence, it appears that the energy required to raise all the stones takes 203 Photons of Planck Length wavelength, and immediately we wonder if this is why the architect divided the truncated height into 203 step layers. Now we have a rationale for the choice of number of steps to build the pyramid, a possible explanation of why this choice was made, and the obvious implication that the builders knew a lot more about physics than we generally give them credit for. Indeed, modern Egyptologists don't even give those ancients any credit for knowing the PI ratio better than, say, $\pi \approx 22/7$, much less granting them knowledge about such advanced details like the modern quantum theory.

Now, it is not known for certain, if the number of step layers in the truncated pyramid was originally 203. It is possible that there were a few more steps, that were removed somehow over the 4500 years the pyramid stood there. But we have a pretty good idea of where the apex of the completed pyramid should be, as Petrie's careful measurement puts this number at, $H = 5776$ inches, so that we can calculate the average implied by the truncation, and then estimate the total number of steps that would be required in the completed pyramid.

$$\text{Avg Step Height} \quad = 5776 \ \text{ inches} \times 15/16 \times 1/203 = 26.67''$$

$$\text{Estimated Number of Steps to Apex} \quad = 5776''/26.67'' \approx 216$$

$$\text{Number of Steps in Capstone} \quad = 216 - 203 = 13$$

That gives us an estimate of about 216 steps to the apex (± 1), of which 203 are there presently in the truncated part, leaving about 13 steps for the capstone that is missing.

Somewhere in the range then, $203 \ \leq \ N \ \leq \ 216$, the architect set his parameters. So, the next obvious question is, how does our analysis change if we change the value for N ? If we discovered, for example, an artist's drawing of the Great Pyramid, done some time in the past, say 1000 B.C., and there were a couple more layers on top of the truncated platform as it stands today, would that change our results significantly?

How sensitive is our analysis to possible changes in what might be considered an uncertain parameter?

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8 A formula for Planck's Constant, h, based on the Great Pyramid's parameters.

We'll look at this problem another way. We recognize that the Energy of the Photon involves Planck's Constant, h, in, $E_s = h \cdot f$. But the frequency, f, and wavelength, $\lambda = c/f$, which appear to use Planck Length, also therefore involve this Planck's

Constant again, and our formulas are thus doubly dependent on this parameter, h. We'd like to isolate the Planck's Constant, and relate it to the dimensions of the Great Pyramid, unpacking it from the double entry accounting that muddles the calculation up somewhat.

So, let's do this first,

$$\text{Planck Length}\ ,\ l_p = \sqrt{\hbar \cdot G/c^3} \tag{15}$$

$$E_s \;=\; \frac{E'}{N} \;=\; \frac{hc}{l_p} \;=\; \frac{hc}{\sqrt{\hbar \cdot G/c^3}} \tag{16}$$

$$E' \cdot \sqrt{\frac{h}{2\pi} \cdot \frac{G}{c^3}} \;=\; N \cdot hc \tag{17}$$

$$(E')^2 \cdot \frac{h}{2\pi} \cdot \frac{G}{c^3} \;=\; N^2 \cdot h^2 \cdot c^2 \tag{18}$$

$$h = \left(\frac{E'}{N}\right)^2 \cdot \frac{1}{2\pi} \cdot \frac{G}{c^5} \tag{19}$$

$$= \left((1 - 61/16^4) \cdot g \cdot \rho \cdot \frac{1}{3} \cdot s^2 \cdot H^4\right)^2 \cdot \frac{G}{2\pi \cdot N^2 \cdot c^5} \tag{20}$$

$$= \frac{1}{2\pi} \cdot \frac{G \cdot g^2}{c^5} \cdot \left(1 - \frac{61}{16^4}\right)^2 \cdot \frac{1}{9} \cdot \rho^2 \cdot s^4 \cdot H^8 \cdot \frac{1}{N^2} \tag{21}$$

So, now we can ask, given that height, H, that slope, s, and that implied density, ρ, how many stone steps, N, must the architect divide the truncated height into if he wishes to convey the idea of the discrete nature of phenomena and record Planck's constant as a measure that characterizes that discretization? Having a set of discrete stone steps already suggests the discretization of the continuous height. But what should the number, N, be, in this case, to perfectly complement that visual suggestion?

Let's now call this encoded estimate of Planck's Constant that depends on N, $h' = h'(N)$, to distinguish it from the true value, h;

With,

$$\begin{aligned}
c &= 299,792,458\frac{m}{s}\ ,\ g = 9.8\frac{m}{s^2}\ ,\ \rho = 2.71\frac{g}{cm^3} = 2710\frac{kg}{m^3}\ ,\ G = 6.6743\times 10^{-11}\frac{m^3}{kg\cdot s^2}\\
H &= 5776 \text{ inches} = 5776'' \times 2.54/100\ \frac{m}{in} = 146.71\ m\ ,\ s = 5.5/7
\end{aligned}$$

we obtain,

$$h' = 2.80677451573\times 10^{-29}\cdot\frac{1}{N^2}\ J.s \tag{22}$$

We know the stone steps are more than 200 and less than or equal to about 216. Currently, there are 203 steps, and if we say that this was the original number, then

$$h' = 2.80677451573\times 10^{-29}\cdot\frac{1}{203^2}\ J.s = 6.811\times 10^{-34}\ J.s \approx 7\times 10^{-34}\ J.s \text{ .. to 1 sig. fig.} \tag{23}$$

$$\text{cf. } h = 6.62607015\times 10^{-34}\ J.s \approx 7\times 10^{-34}\ J.s \text{ .. to 1 sig. fig.} \tag{24}$$

if we required, $h' \equiv h$, then , $N^2 = 2.80677451573\times 10^{-29}/6.62607015\times 10^{-34} = 42,359.57$, and, $N = \sqrt{42,359.57} = 205.8$ stone steps. Since the steps are discrete, the closest would be to build 206 stone steps for the truncated pyramid. Then,

$$h'(N=206) = 2.80677451573\times 10^{-29}\cdot\frac{1}{206^2}\ J.s = 6.614\times 10^{-34}\ J.s \tag{25}$$

In general,

$$h'(N)/h = \frac{2.80677451573\times 10^{-29}}{6.62607015\times 10^{-34}}\cdot\frac{1}{N^2} = \frac{42359.57}{N^2} \tag{26}$$

Let's examine more closely how the Planck's Constant encoding varies with the number, N, about these values. We'll use our modern units of $J.s$, with which we're familiar, and also natural units in which we effectively set, $h = 1$, to eliminate the special nature of our definitions of Joule for Energy and Second for Time, so that we can compare the numbers in a manner independent of the particular metrological system employed to record and view the numbers. So, we divide the estimate, $h' = h'(N)$, by the true value, h, to write the variable precision Planck's Constant parameter found there as a dimensionless ratio, h'/h, to isolate just the effects of different precision requirements on the selection of N.

$$1 \pm 0.05, \quad 1 \pm 5/100, \ \ or \ \ 1 \pm 1/20$$

$$1 - 1/20 \ \ < \ \ h'/h \ \ < \ \ 1 + 1/20$$

$$\text{(J.s)} \qquad\qquad\qquad\qquad\qquad \text{(h=1)}$$

$$\begin{array}{rcl}
h'(N = 200) = 7.0169 \times 10^{-34} \ J.s & \rightarrow & 1.058989 = 1 + 0.058989 = 1 + 5.8989 \times 10^{-2} \\
h'(N = 201) = 6.9473 \times 10^{-34} \ J.s & \rightarrow & 1.048478 = 1 + 0.048478 = 1 + 4.8478 \times 10^{-2} \\
h'(N = 202) = 6.8787 \times 10^{-34} \ J.s & \rightarrow & 1.038123 = 1 + 0.038123 = 1 + 3.8123 \times 10^{-2} \\
h'(N = 203) = 6.8111 \times 10^{-34} \ J.s & \rightarrow & 1.027920 = 1 + 0.027920 = 1 + 2.7920 \times 10^{-2} \\
h'(N = 204) = 6.7445 \times 10^{-34} \ J.s & \rightarrow & 1.017867 = 1 + 0.017867 = 1 + 1.7867 \times 10^{-2} \\
h'(N = 205) = 6.6788 \times 10^{-34} \ J.s & \rightarrow & 1.007961 = 1 + 0.007961 = 1 + 0.7961 \times 10^{-2} \\
h'(N = 206) = 6.6141 \times 10^{-34} \ J.s & \rightarrow & 0.998199 = 1 - 0.001801 = 1 - 0.1801 \times 10^{-2} \\
h'(N = 207) = 6.5504 \times 10^{-34} \ J.s & \rightarrow & 0.988578 = 1 - 0.011422 = 1 - 1.1422 \times 10^{-2} \\
h'(N = 208) = 6.4876 \times 10^{-34} \ J.s & \rightarrow & 0.979095 = 1 - 0.020905 = 1 - 2.0905 \times 10^{-2} \\
h'(N = 209) = 6.4256 \times 10^{-34} \ J.s & \rightarrow & 0.969748 = 1 - 0.030252 = 1 - 3.0252 \times 10^{-2} \\
h'(N = 210) = 6.3646 \times 10^{-34} \ J.s & \rightarrow & 0.960534 = 1 - 0.039466 = 1 - 3.9466 \times 10^{-2} \\
h'(N = 211) = 6.3044 \times 10^{-34} \ J.s & \rightarrow & 0.951451 = 1 - 0.048549 = 1 - 4.8549 \times 10^{-2} \\
h'(N = 212) = 6.2450 \times 10^{-34} \ J.s & \rightarrow & 0.942497 = 1 - 0.057503 = 1 - 5.7503 \times 10^{-2}
\end{array}$$

$$\pm 0.050000 \quad .. \pm 5.0000 \times 10^{-2} \quad i.e. \pm 1/20$$

1 part in 20, above or below the value

Now the architect has to decide what will be a meaningful precision to encode the value of Planck's Constant. Here is where we assume his numbers are in a system like ours, based on the decimal powers of 10, so that he's looking for the significant figure in the difference to be within the midway digit, that is, within ± 5, or else the figure might be rounded up or down to another place in the decimals, and throw the viewer off the count. For example, $1.058989 \rightarrow 1.1$, and, $0.942497 \rightarrow 0.9$, where we'd really like, `x.xxxxxx` $\rightarrow 1.0$, in the table shown. As we can see from the above table, this is achieved by doing better than 5 parts in 100, i.e. $\pm 5/100$, or, which is the same thing, 1 part in 20.

Unraveling this result, $h'(N = 203) = 1.027920 \cdot h = 6.81 \times 10^{-34} \ J.s.$, back into our modern units, we see the order of magnitude $10^{-34} \ J.s.$ is there, along with a number that rounds to 7 as the significant digit, and also truncates to 6, in similar manner to the true value of Planck's Constant, $h = 6.626 \times 10^{-34} \ J.s.$ And thus the leading significant figure is unambiguously represented, along with the order of magnitude, when back in our own units. We don't know what units the ancient architect might have used for Energy and Time, but as long as the number system used was the decimal base-10, and not say, the sexagesimal base-60, we'd expect the numerical considerations to be akin to our modern treatment of significant figures and follow a similar argument as we have done here.

This $1/20$ precision theme is encoded in the dimensions at the base of the Great Pyramid as the number of parts the length L is to be subdivided. The proper discussion of this is long, beyond the scope of this short paper, and will be dealt with elsewhere. Suffice it to say, that the base at the Great Pyramid is not an exact square, but deviates from that perfect rectilinear form, and the specific deviation encodes the units of length that must be used to interpret the structure. This leads to one of the main results, that L is to be divided into 20 parts, and thus the unit to be applied in this case is 1 part in 20 of that length. This theme is repeated again in the King's Chamber, whose variances reveal once more that the East Wall is to be divided into 20 parts there also. So, the idea meets with Petrie's criterion that any theory of the pyramid plan must exhibit 'repetition' in some form in other parts of the structure. This reminds us of an Old English relationship of units, where the length called the '*ell*' was divided into 20 parts measured by another unit called the '*nail*'. The *nail* was $1/20$ of an *ell*, but $1/16$ of a *yard*. For now, we just note that by setting 1 part in 20 as the limit for the precision, when using natural units ($h = 1$), we find the number of stone steps must be greater than, $N = 200$, and less than, $N = 212$. The architect is then free to pick any number in this range, allowing him the flexibility to encode other things simultaneously in the monument by optimising the choice of N in that range. So, for example, $N = 203 = 7 \times 29$, so the current step count is divisible by 7, just as the royal cubit standard measuring rod is evenly divisible into 7 palms, and being divisible by the number 7, as Petrie points out, is one of the repeating themes used throughout the pyramid.

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9 2π or not 2π ?

Planck Length is a somewhat civilization-independent measure that mostly depends on fundamental constants of nature. The idea behind it, is that even aliens across the galaxy would be able to understand and communicate using measurements reported in numbers of Planck units. Those aliens wouldn't be expected to grasp the meaning of units based on our personal body dimensions, like the foot, the inch, and the yard, found in the Imperial British system; nor based on our earth's dimensions, like the metre, which aims to represent the length of about one ten thousandths part of a quarter of the Earth's circumference on a great circle running through the French city of Paris and the poles (since the Earth is not a perfect sphere we have to specify which great circle to use!); or a certain length of a special bar made of platinum-iridium alloy stored in that city of Paris, which was decided on at committee meetings, and so on. These are all special units unique to our local environment, which we just made up ourselves, that are convenient for us, that require the interpreter to know quite a bit about our local environment, customs and practices, before "getting it," if they received a message from us containing such measurements.

In 1899 Planck suggested$^{(13)}$ that the fundamental constants, $\{c, G, h\}$, which are dimensioned parameters, $[\,c\,] = \mathsf{L/T}$, $[\,h\,] = \mathsf{L^2\ M/T}$, $[\,G\,] = \mathsf{L^3/(MT^2)}$, consisting of the three basic dimensions of mass (M), length (L), and time (T), could be combined in formula expressions to construct an alternative set of units of measure that would then be independent of our somewhat arbitrary conventional systems of civilization-influenced metrology. The three fundamental units could be expressed by the formulas;

$$\text{unit of length} = \left(\frac{hG}{c^3}\right)^{1/2} \quad , \quad \text{unit of time} = \left(\frac{hG}{c^5}\right)^{1/2} \quad , \quad \text{unit of mass} = \left(\frac{hc}{G}\right)^{1/2}$$

Presumably, those aliens across the galaxy would find the speed of light (c), the gravitational constant (G), and the energy quantization constant (h)–which we call Planck's Constant in our civilization–all turn out to be also constants in their own scientific experiments and match up with our values given the appropriate conversion tables between the two unit standards, theirs vs ours. It would be just as obvious to them, however, as it is to us, that a civilization-independent set of units could be constructed from the same three, $\{c, G, h\}$, and could therefore be used before first contact, so that we would not need to have a previously established relationship with the aliens to obtain that inter-civilization conversion table beforhand to decrypt the messages going back and forth, in order to read each other's messages.

This same thing would be even more true if a civilization in the remote past wanted to send a message down through time to another civilization in the distant future. The civilization-dependent practices of measuring things would change over time in unknown ways, but these special units of length, time, and mass, based on fundamental constants of nature would remain the same. Since the ancient civilization would no longer exist, there would be no way for the future civilisation to establish that interconversion table without inventing time travel, unless they discovered some objects that held the ancient units in their measures that could be re-measured in the future. But, what if the time difference is so long, between past and future, that all the usual artifacts of the ancient world decay and disintegrate before the future archaeologists got a chance to examine them? The ancients would have to build an artifact so huge and stable that it would not only be easily found, but would likely still be standing, surviving largely intact, to give the future metrologists a chance to get the message.

It would be natural, then, for the ancients to put these fundamental constant derived units, which we have given the title "Planck Units," today, into their monument, somehow, and we should not really be surprised to find one or more of these special units there.

The Great Pyramid then appears to be an obvious giant ancient "Time Capsule," containing a message bottled up and written in the one and only universally known general civilization-independent language: that is "geometry;" therefore, finding a piece of scientific data encoded in a similar civilization-independent scientific metric, such as "Planck Length," in part of that message, should not really be unexpected.

Indeed, by making the "Planck Length" so obvious from just a cursory examination of the external features, it suggests that this is just the enticement, the advert on the label on the Capsule, encouraging the visitor to open up the bottle and see what else might be inside! Well, the Caliph "Al Mamun" physically opened the bottle in 820 A.D., but he was looking for a treasure of "gold," while the advert on the label on this bottle clearly indicates, now that we can apparently read it, the treasure is "scientific information." Al Mamun couldn't find this treasure, because his mind was not suitably prepared, as the message was not intended for him, but for those who would come afterwards; those who could see and understand ideas like "Planck Units." This giant artifact is then a stone book encoded in mathematics, so its treasure is hidden from those without eyes to see.

But, one apparent problem with this Planck system, is that such units are only obviously unique up to an arbitrary multiplicative dimensionless factor that still initially looks as if it depends on our convention.

Should we use h or $h/(2\pi)$?

$$\text{unit of length} = \sqrt{\frac{h}{2\pi}\frac{G}{c^3}} \quad \text{- or -} \quad \text{unit of length} = \sqrt{h\frac{G}{c^3}}$$

While Planck's original set of units used h, it has become the fashion to replace that with $\hbar = h/(2\pi)$, instead, and to report the cycle frequency, f (Hz), in terms of the equivalent angular frequency, $\omega = 2\pi f$ (rad/s). In which case, corresponding to the period, T (s) $= 1/f$(Hz), we have an angular period, $\tau = T/(2\pi) = 1/\omega$. Today, the most widely used and reported definition of the Planck Length, l_p, is therefore,

$$l_p = \sqrt{\hbar\frac{G}{c^3}} \tag{27}$$

Of course, when we use, $\hbar = h/(2\pi)$, we get a "smaller" size for the Planck Length. So, in a sense, we could say that including the "2π" here gives a better unit of "tiny scales" to measure things, at least as far as the "length" parameter is concerned. There's a good reason to occasionally include the factor of "2π" in Planck's constant, and so use, $\hbar$, instead of h, because it often simplifies some computational work, but not always. Sometimes the standard form, h, is just more convenient to use.

The underlying cause for the "2π" ambiguity, that appears in these unit definitions, is the principal role "geometry" plays in constructing models of phenomena. We think in terms of geometry when constructing descriptions of the world, and two of the simplest geometric forms that exist, that are employed in this way, are the circle and the sphere. These geometric objects have the maximal symmetry possible for the 2-dimensional plane and 3-dimensional space, respectively. When we model phenomena, as a first approximation, we use the simplest forms we can, and then we add onto that various permutations, perturbations, and higher order constructs to fit the model better to the measured reality. The geometric space circle is conveniently adapted to describe periodic time cycles, to give us a way to visualize time, and relate it to the more easily comprehended space length measure. Similarly, the geometric space sphere is used to describe phenomena that have no preference for direction about some point, like the force field emanating from a charge or mass. Just like the length of a circle perimeter is $2\pi r$, the surface area of a sphere is $4\pi r^2$, and when it comes to describe the Gravitational force, there's another choice of including "4π", or leaving that out, in the definiton of the Gravitational Constant. We often do this for the Coulomb Electric Force, for example:

$$|F_g| \quad = G \cdot \frac{M_1 \cdot M_2}{r^2} \quad = G' \cdot \frac{M_1 \cdot M_2}{4\pi r^2} \quad \text{.. Where } G' = 4\pi G \text{ .. Newton} \tag{28}$$

$$|F_e| \quad = k_e \cdot \frac{|q_1| \cdot |q_2|}{r^2} \quad = \frac{1}{\epsilon_0} \cdot \frac{|q_1| \cdot |q_2|}{4\pi r^2} \quad \text{.. Coulomb} \tag{29}$$

The $4\pi r^2$ captures that 3-dimensional geometric idea of there being a "flux" through the surface of the sphere. The total flux is the same, regardless of the size of the radius, r, so then the flux per unit area at any point on the surface of the sphere decreases as the radius becomes larger, and then logically the effect on a point particle at the surface will fall off as $1/r^2$. We could recognize this 3-dimensional geometric idea by writing, $G' = 4\pi G$, putting in, or extracting out, that geometrically determined fixed dimensionless factor of "4π", to include or separate the pure geometry modeling framework, used to interpret phenomena, from the pure physical factor in defining the gravitational constant.

So, there's another ambiguity in the definitions of these units using fundamental constants, as we could decide on either of the two, G or $4\pi G$, just like we had a choice of, h or $h/(2\pi)$. Then, other considerations introduced by more advanced theories of Gravity, like Einstein's General Relativity, which turns more of that physical effect into a pure geometric model, by explaining gravitation as the curvature of a 4-dimensional spacetime, then suggest perhaps, using, $8\pi G$, might simplify things best.

Luckily, so far as we know, however, this is the only ambiguity we have to deal with in defining a universally accepted unit of length that is both obvious and independent of the usual arbitrary ways of establishing dimensioned measurements for physical parameters. We just have to decide whether to include the "π's" or leave them out. What would the aliens in another galaxy do? Is there any logic that would suggest one is better than the other? How about just simply picking the smallest of the lot? After all, we measure larger things with smaller units, in order to count conveniently in whole numbers when possible, and avoid fractions. So, picking the smallest unit seems logical, from a purely practical point of view.

The geometric modeling employed in the frameworks of understanding therefore introduce certain factors of π, specifically, 2π, 4π, or 8π, for 2-D, 3-D, or 4-D modeling, giving us alternative possible conventions dependent on, h or $h/(2\pi)$, and, G or $4\pi G$ or $8\pi G$. But, among these alternative variations, the current choice for the Planck Length is still the smallest unit of all possible constructions we could adopt by convention, i.e.;

$$l_p = \sqrt{\frac{h}{2\pi}\frac{G}{c^3}} < \sqrt{h\frac{G}{c^3}} < \sqrt{\frac{h}{2\pi}\frac{4\pi G}{c^3}} < \sqrt{\frac{h}{2\pi}\frac{8\pi G}{c^3}} < \sqrt{h\frac{4\pi G}{c^3}} < \sqrt{h\frac{8\pi G}{c^3}}$$

Being the smallest of the list, we could then also say it's the only convention-independent one, since it's not smallest by our convention, it's just the smallest of all conventions that could be logically constructed, given our geometric visualization of phenomena.

It is curious then, that the Planck Length encoded in the Great Pyramid is also the smallest one, with, $h/(2\pi)$ and G. So maybe the ancients used that logic in selecting the the value, or maybe they knew something that we have yet to discover, and which we are only now contemplating, so that among our various speculations the one that will turn out correct will be that one matching the very same found encoded in the monument. Maybe it's not just a convenient civilization-independent tiny scale, but the limit of the lengths that can be manifested in the universe. We can only speculate, since current experimental knowledge limits us to length scales with size of about $10^{-22}\ m$, and longer. To get down to $10^{-35}\ m$, we have a long way to go. But, in 1964, C. Alden Mead noticed that our current theories of physics will have trouble explaining things when we get down to the length scales on order of Planck Length$^{(14)}$, because then the Gravitational effects of interacting particles can no longer be ignored; Gravity and Quantum Phenomena will now be operating in the same playground, with equally non-negligible effect on these tiny scales, and we have no proper Quantum Theory of Gravity to tell us what would happen in that domain. Hence, the suggestion is that the Planck Length may either be a fundamental limit of the universe, or at least the limit to our current theories of physics ability to explain and predict what would happen in the neighborhood of that tiny domain.

An 8-Dimensional Geometry?

We notice that the Great Pyramid's formula for Planck's Constant, h, seems to suggest to us that this quantum mechanical constant derives from the geometry of an 8-dimensional spacetime. The selectable design parameter available to the architect is the linear dimension of the pyramid, which we've represented as the height to apex, H, and the fomula exhibits the dependence, H^8, which we can also re-write, $(H/c) \cdot (H^7)$, by factoring out one of the light speed constants, c, from that expression. We then find,

$$[\,h\,] \sim [H/c \quad \cdot \quad H^7] = \mathsf{T} \cdot \mathsf{L}^7$$

The other factors in the formula are constants, apart form the selectable integer, N, for number of steps in the truncated height; But N is a dimensionless factor. So, if given, G and g, then, understanding that Calcite has only this one definite density, $\rho = 2.71 g/cm^3$, and the slope of the pyramid, s, is fixed by the convergence of the geometries crossing at that particular slope value, what is left to the architect to fit to the quantum constant is the linear height, H. In this way, we can see the hint being given to the viewer, that to understand exactly how that quantization constant, h, arises in nature, we first have to master the 8-D geometry, with 1 Time and 7 Space dimensions, which is essentially the geometry that corresponds to something like Octonion Algebra. We already have Einstein's General Relativity which gives a 4-D geometric model of Gravity, explaining gravity as the curvature of spacetime, but the Pyramid seems to be telling us we need an 8-D geometric model to bring Gravity and Quantum Phenomena together into one neat model. At that point, both Gravitation and Quantum Phenomena will have purely geometric models, and can then be fit together like parts of a puzzle.

A principle of Equivalence !

While we're at it, geometry is the language of the pyramid. But, this Great Pyramid introduces us to a particular idea using geometry: the "equivalence," using geometric forms. Those geometries that cross at the junction, represented by the base angle or slope, are really exploiting the idea that one thing is equivalent to another thing, in some way, despite difference in appearence.

First, we have an expression in 1-dimensional space measures, where the rectilinear *length* of the perimeter of a square is equivalent to the curved *length* of the circle's circumference. Second, we have John Taylor's observation, in a 2-dimensional space measure, that the *area* of a triangle is equivalent to the *area* of a square. Third, we have the cube model, an expression in 3-dimensional space, that suggests the *volume* of the truncated pyramid is equivalent to the *volume* of a cube.

Three equivalences in one shape! A 1-D, a 2-D, and a 3-D equivalence principle, all at the same time, meeting together and fixing the slope of the structure. The repeating of this theme suggests that we take note of 'equivalences' between things that otherwise look different. Well, we know Einstein did just that. Twice in fact. First, Einstein observed that the length traveled by a ray of light in some unit of time in one inertial frame of reference, is equivalent to the length light travels in any other inertial reference frame in the unit of time there, no matter how fast the two frames are moving towards or apart from each other. All observers would compute the same speed for light when they computed their ratio of length/time. This was Einstein's first equivalence principle, which gave us Special Relativity. Then, for a long time people wondered about the meaning and interpretation of mass. On the one hand, mass is a quantitative parameter that represents the "resistance" a body puts up when a force acts on it attempting to move it along. This was called "inertial mass," because the idea of "inertia" is "resistance" to change. But, then the presence of mass also appears to "cause" things around it to move, exhibiting it's own tendency to make other things change whenever they come near. This was called "gravitational mass," and seemed to be quite a different type of thing from "inertial mass." The two apparently produce opposite effects, one "causing change" and one "resisting change." But, no matter how hard scientists tried to construct experiments to detect a different value for the mass parameter, using these two

observations, they couldn't detect any difference in the value. So, Einstein said–taking a leap of faith, from the inability of scientists to find a measurable difference in the two conceptually different things–the reason why nobody can measure any difference, is because they are really 'equivalent', despite the fact that they look totally different from the effects they are observed to produce–and consequently one cannot tell the difference "locally" between the effect of gravity acting in an *inertial* reference frame from a that where there's no gravity but the observer's reference frame is *non-inertial* accelerating relative to the observed body. This is Einstein's second Principle of Equivalence, which eventually led to the development of his General Theory of Relativity. And here we are at the Great Pyramid, which is telling us "equivalence" is the key, pay attention to it. Maybe there are more equivalences to be found, so if we stay on that theme we may uncover the secrets of the universe that all scientists are on the hunt for. The one thing the Pyramid seems to add to our conceptual development of models of the universe is the "simultaneous equivalence" of several n-dimensional metrics working together at the same time to bring about the emergence of phenomena in our reality.
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10 Conclusion.

We have seen a simple energy calculation that suggests that something very close to the Planck Length is found as implicated in the Great Pyramid; which lets us calculate a reasonable estimate of Planck's Constant, h, using the standard formulas of Physics. This sets the range of stone steps the architect could use to encode this parameter in the structure to between 201 and 211, inclusive, in order to achieve at least 1 part in 20 precision in the natural units representation of the value, where the true value is taken as the unit, $h = 1$.

This calculation assumes a missing capstone of order $1/16^{th}$ of the Pyramid's dimensions be taken into account, as found in the actual truncated Pyramid at Giza.

From the average height of a stone step, we can infer that, although only 203 steps are present in the truncated structure as it stands today, the intended suggested number of steps to the apex if completed, while maintaining that average height, would be something close to 216. These numbers have their own alternative significance in both mysticism and mathematics. The number 216 is the perfect cube–a number that refers to reincarnation in the Greek number mysticism introduced by Pythagoras and followed by Plato, which was in turn presumably inherited from the Egyptian Priests when Pythagoras went to Egypt to seek knowledge from the old world. It is the cube of the Perfect Number 6, which is said to be Perfect because it's the sum of its proper divisors, $6 = 3 + 2 + 1$; the second such perfect number being 28. The number 203 is divisible by 7, a number long considered sacred in the design of the world. For example, the forearm of a man is 7 palm widths long, from elbow to longest fingertip, where the palm width is measured just across the base of 4 fingers excluding the thumb, and this gives us the royal cubit length, as $7 \times 4 = 28$ digits long; a number which also matches the number of finger segments that man has on his two hands: 14 on the left hand and 14 on the right hand, making 28 finger segments; and these are, of course, the tools he uses both to measure and to fashion and create things, to make artifacts, like the pyramids. Also, the number 203 can be expressed as the sum of 7 squares. Moreover, the number 203 is both the sum of 3 squares and the sum of 7 squares.

There are exactly 14 squares less than this number and they all sum to 5 times the number itself:

$$1+4+9+16+25+36+49+64+81+100+121+144+169+196=1015=5\times 203$$

like our 5 sided pyramid with 203 square steps–*and signature of 5-fingered man.*

Remarkably, the square of the angular period of a pendulum oscillating at Giza, with length equal to that truncated height, H', is $14\ s^2$, suggesting that the unit of the 'second' is indicated in the pyramid design by exploiting that very coincidental numerical fact of there being 14 squares "below" the number 203, just as the pendulum bob swings "below" that truncated platform. In the language of geometry, the square is a type of form, immediately associated with a square numbers in arithmetic, and so a number of geometric shapes of the "same type" is being employed suggestively, for a number of particular square "units" of a kind.

$$\tau^2=\frac{H'}{g}=5776\ in\times\frac{15}{16}\times 2.54\ \frac{cm}{in}\times\frac{1}{100}\frac{m}{cm}\times\frac{1}{9.8\ \frac{m}{s^2}}=14.0\ s^2 \tag{30}$$

The *second* represents time on the order of the periodic beat of the human heart, a unit otherwise conveniently represented, and given a more precise and less variable value, by dividing the day cycle into an even number by 24 hours, the hour into 60 minutes, and the minute into 60 seconds–one of the remaining vestigial metrological ratios of time inherited from the ancient Babylonian sexagesimal system.

Significantly, we can break parts of this 14 squares sum out into the partial sub-sums:

$$203=1+81+121 \qquad =1^2+9^2+11^2 \tag{31}$$
$$203=4+9+16+25+36+49+64 \qquad =2^2+3^2+4^2+5^2+6^2+7^2+8^2 \tag{32}$$

which then reminds us immediately of the algebras of Quaternions and Octonions, respectively,

$$-3 \qquad =i^2+j^2+k^2 \qquad \text{.. Quaternions} \tag{33}$$
$$-7 \qquad =e_1^2+e_2^2+e_3^2+e_4^2+e_5^2+e_6^2+e_7^2 \qquad \text{.. Octonions} \tag{34}$$

as these two hypercomplex algebras are similarly the sums of 3 and 7 squares of the units making up their dimensional space. Once again, the "shapes of a type" represent "units of a kind." The '-3' or '-7' on the L.H.S. typically represents the 'time' dimension, and counts the number of space dimensions enfolded into the root of time for that algebra, just as the 203 is the count in the vertical direction, and the vertical is automatically associated with time; i.e. a stone falls vertically under gravity, and takes 'time' to drop one length to another length, and the length it travels is proportional to the 'square' of the time; $H' = 1/2 \cdot g \cdot t^2$; so that the drop length is indeed made up of 'squares', and so on, just as the pyramid height is made up of 203 'squares' being the shape of the stone layers. Notice that the square of the time it takes a stone to fall to the ground from the truncated height, H', is $28\ s^2$, just double that of the pendulum's angular period square;

$$t^2 = 2 \cdot \frac{H'}{g} = 28\ s^2 \tag{35}$$

But once one has recognised the presence of the unit of the 'second' in the pyramid's design, by measuring the 'time swing of a pendulum' and the 'time of drop of a stone' to the ground at Giza, matching those up to the numerical indicators, 14 and 28, both from that truncated height, H', which are the keys to time–artfully encoded by choice of numerical parameters in design–it then becomes obvious that the 'speed of light' is also encoded in the base of the pyramid in units of length per second, since the difference in circumference lengths, between the inscribed and circumscribed circles, of that base square, is just the distance light travels in one second, on the scale of $10^6 : 1$ (10^6 is the highest value numeric glyph in the Ancient Egyptian Hieroglyphic system..a scaling factor used here to encode the highest speed anything can travel!). Now,

$$\text{Base Square Length} = 2L \tag{36}$$

$$\text{Inscribed Circle Circumference} = \pi D = \pi(2L) \tag{37}$$

$$\text{Circumscribed Circle Circumference} = \pi D' = \pi\sqrt{2}(2L) \tag{38}$$

$$\text{Difference in Circumferences} = \pi(2L)(\sqrt{2}-1) \tag{39}$$

$$\text{Distance Light Travels in 1 Second} = 1s \cdot c = 10^6 \times \pi(2L)(\sqrt{2}-1) \tag{40}$$

$$.\!: \text{ with, } c = 299,792,458\ m/s \tag{41}$$

$$(2L) = \frac{1s \cdot c}{10^6 \cdot \pi(\sqrt{2}-1)} = 230.381\ m \times 100\frac{cm}{m} \times (1/2.54)\frac{in}{cm} = 9,070'' \tag{42}$$

Survey data:

Petrie 1883:	N.side $9,069.4''$	E.side $9,067.7''$	S.side $9,069.5''$	W.side $9,068.8''$	Mean $9,068.8''$
Cole 1925:	N.side $9,065.1''$	E.side $9,070.5''$	S.side $9,073.0''$	W.side $9069.2''$	Mean $9,069.4''$

This is the side length of the base of the pyramid at the casing square perimeter, i.e. the line where the tura limestone casing meets the pavement on which the pyramid stands. Notice that the PI-model–which indicates the geometry of a circle circumference being equal to the square perimeter, with height as radius–gets us thinking about *circles* constructed at the base of the pyramid. Then, when we look inside the square, and outside the square, for those inscribed and circumscribed circles, we discover the speed of light, and the rationale for setting the particular dimension of length for the base of the pyramid. So, the PI-model is the 'hook' that turns our attention to the idea of considering circles at the base. Also, the relation between the angular period, τ, and the period, T, is $T = 2\pi\tau$, which again is hinted at by the PI-model's height (τ) to perimeter (T) geometry.

Lest anyone think that the speed of light found there is just accidental, notice that the architect employs "two" circles to encode the value. The language here is geometry, and the designer has to use static geometric imagery for dynamic things. Well, since James Clerk Maxwell published his 1873 *Treatise on Electricity and Magnetism*, we've known that light, in fact, is made up of "two" oscillating parameters, an Electric Oscillation and a Magnetic Oscillation, travelling together. An oscillating parameter is represented geometrically by the plane circle, as this is the most natural visualization of something repeating and returning to the same point again and again. Those two circles, inside circle and outside circle, are then seen as representative of the idea that this "light," whose speed is being encoded, is a phenomenon that possesses "two" oscillating parameters, Electric and Magnetic, in its composition. The Electric would naturally be the inner circle, and the Magnetic the outer circle, since atoms are mostly neutral, hiding their electric nature (i.e. inside the square), but their magnetic effects extend well beyond the space they occupy, to the outside world, causing effects far beyond the atomic limits (i.e. outside the square). Those base circles are also at right angles to the translational movement, as exhibited by the path a stone falls, i.e the vertical quantization "h" axis.

The one apparent paradox, of course, with this observation, is that we already pointed out the architect only has one parameter free to fit Planck's Constant, h, to the pyramid design, and this was a linear dimension of the pyramid. We picked the height, H, to show how it could be set along with the step count, N, to fit h. But, in order to fit the speed of light, c, using any linear dimension of the same pyramid, there must be an unknown law of physics, or a marvelous coincidence of nature, that would permit both "c" and "h" to submit to the architects design, that he could encode both of these constant parameters there. Of course, we do not fit these parameters to a very high level of precision, since, h, for example, is only encoded to one significant figure, and, c, also, would be considered accurate or less so, depending on how well we eventually manage to measure the base of the pyramid. Moreover, if we accept we are allowed to "switch models," as the pyramid is being viewed only as our "instructor," directing our attention to this and that, for consideration, we could get the precision level as high as we'd like, by fine tuning the particular model that fits best. The marvel, however, is still that these things are close enough together, that a single geometric monument like the Great Pyramid could be designed and constructed to implicate all these parameters with sufficient precision that we can recognize them from our own scientific knowledge.

Another interesting observation, is that, while the North side of the Great Pyramid has 203 steps, the South Side only has 201 steps; these numbers are evenly divisible by 7 and 3, respectively, as, $201 = 3 \times 67$, and, $203 = 7 \times 29$; so we have again that paring of "3" and "7" together, which are not only suggestive of Quaternions and Octonions, but are also the first two elements in the continued fraction expansion for $\pi = [3; 7, 15, 1, 292, ...] \approx [3; 7, 16]$; hinting at the idea that part of the message in the Great Pyramid is to inform of something that repeats time and time again.

So, if the architect is trying to pack all this information loaded with hints into the few parameters available in his monument, with only "numbers" and "shapes" to carry the message, he has to go about it in a somewhat clever way, and make tradeoffs here and there, while still managing to get his message across. The apparent objective of the design of the pyramid is to make the viewer "think" about what is found there; selecting certain particular things out of the vast number of possibilities that one could explore on one's own, so as to guide the mind of the viewer back onto those few things that he should consider first, if he wishes to find the answers he is looking for; like the popular dictim says, "*Seek and ye shall find.*" But, look here first.

We can imagine the architect selected his number with a partial algorithm something like this:

1.. exactly 14 squares below N	:	$197 \leq N \leq 224$	..hints at the unit of the 'second'
2.. 1/20 precision for h'/h	:	$201 \leq N \leq 211$	..good enough to recognize h
3.. 1/40 precision for l'_p/l_p	:	$201 \leq N \leq 210$	..excellent to recognize l_p
4.. divisible by 7	:	$196, 203, 210, 217, 224$	..hint at sacred number 7
5.. satisfying 1,2,3, & 4	:	$203, 210$	..all of the above constraints
6.. closest to $h'(206)$, etc..	:	203	..best pick for maximum impact

Some advantages of picking, $N = 203$, are that we not only get Planck's Constant encoded with a meaningful precision, and suggest division by 7 as an important design parameter, but at the same time are able to hint at the algebras that likely underly the very structure of the universe. All these things become obvious to the visitor of the Giza plateau, once he starts to contemplate the numbers.

If this is all just a coincidence, it is a marvelous coincidence indeed. Occam's razor suggests, however, that the simplest answer to the question of "How could this be?", is that the architect designed it this way by deliberate intent. It is clear that several natural coincidences are to be found in the pyramid's construction. But to think that the architect did not know any of these coincidences when designing the monument, and did not deliberately employ some of them for effect in his work, but that he went about blindly and totally unaware of these things, and the resulting consequence of his random choices for parameters is that they just so happen to accidentally fall neatly and exactly in the places where the coincidences collide, defies credulity. So, we conclude that we've probably identified here one or more of the reasons for choice of, $N = 203$, for the number of stone steps in the Great Pyramid as found in that monument today. In the base we have "c", going around in a pair of circles, like an oscillating pair of field vectors, and in the steps we have "h", measured from the discrete jumps of a continuous height parameter; so the geometric imagery employed is in perfect harmony with the ideas we have about the very parameters whose numerical values are discovered there, and it's not just a random occurrence of some numeric fluke that a critic could label mere numerology. It's not just about the numbers, it's what the shapes also telegraph to us, that make the numbers matter.

! All calculations performed with Swiss Micros DM42 High Precision Calculator to 34 significant figures.

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APPENDIX A

[QUOTE]

#.107. "PYRAMID" THE GRECIANISED FORM OF HEBREW . "URIM–MIDDIN"—"LIGHTS–MEASURES."

. . .
. . .
For the name "Pyramid"—like all the constructional conceptions embodied in the Great Pyramid—is primarly of Euphratean derivation. The Egyptian name for the Great Pyramid is "Khuti"—"the Lights." In the Semitic languages the equivalent name is "Urim"—"the Lights." In Phrygian and Greek, the root "Ur" (light) became sucessively "Pur" and "Pyr" (fire), and "Pyra" (a plural) "beacon fires."
. . .
. . .
. . .
In Chaldee and Hebrew, 'Urim' = 'Lights,' with the labial 'p,' as in Phrygian, 'Urim' = 'Purim' = 'Lights.' Greek 'Pyra' = 'beacon fires' (lights).

In Chaldee and Hebrew, 'middin' = 'measures.' Hence the Chaldee-Hebrew name for the Great Pyramid—in Egyptian, 'Khuti,' 'the Lights'—is 'Urim-middin' (Purim-middin)=' Lights-Measures.'

In Greek this becomes 'Pyra-midos,' 'Pyra-mid.'

The name 'Pyramid,' therefore, monumentalises the external purpose of the Great Pyramid. It is a "beacon of reflexions," and a "monument of measures."

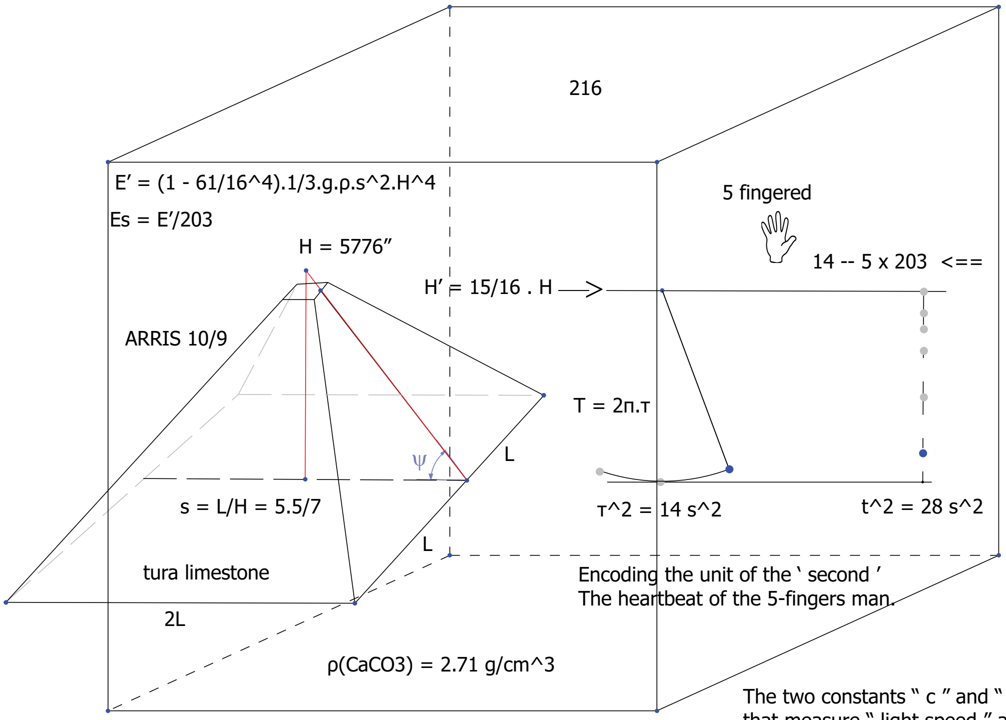
It is of importance to observe that the Greek name is not derived from two Greek roots, but is the Grecianised form (Pyramidos) of the Semetic name 'Urim-middin,' or possibly of a composite Phrygian and Phoenician form of the name—'Purim-middoh.'

[END QUOTE.]

Quote taken from:

Davidson, D., and Aldersmith, H., "*The Great Pyramid its Divine Message.* AN ORIGINAL CO-ORDINATION OF HISTORICAL DOCUMENTS AND ARCHAEOLOGICAL EVIDENCES.", Kessinger Publishing Company, Montana, U.S.A., ISBN 1-56459-116-6.; pp.86-87.

A MONUMENT OF MEASURES



01 -- 001
02 -- 004
03 -- 009
04 -- 016
05 -- 025
06 -- 036
07 -- 049
08 -- 064
09 -- 081
10 -- 100
11 -- 121
12 -- 144
13 -- 169
14 -- 196

The two constants ‟ c ″ and ‟ h ″
that measure ‟ light speed ″ and ‟ light quantization ″
encoded in the Great Pyramid of Giza

Planck Length: l_p = hc/Es = 1.6 x 10^-35 m

Planck’s Constant: h = (Es)^2 . G/(2п.c^5) = 7 x 10^-34 J.s

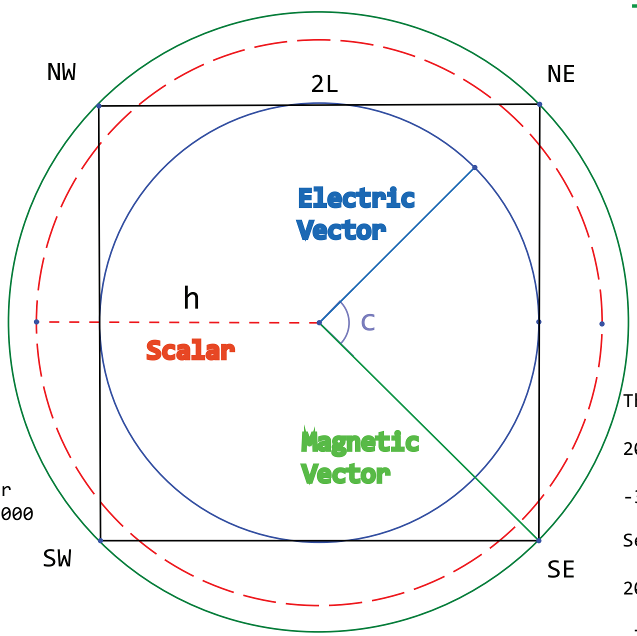
speed of light: c = 10^6 x п (2L) (√2 - 1) x 1/(1 second) = 2.9979 x 10^8 m/s (when, 2L = 9070″) o(| PI - [3;7,16] |) ~ 1/10^6

THE LIGHT MEASURES

Numeric
Hieroglyphs

10^6
10^5
10^4
10^3
10^2
∩ 10
▯ 1

Highest glyph for
number is 1,000,000



THE GREAT PYRAMID BASE SQUARE

Base Angle
Models and Data

PI-model: Ψ = 51;51;14
EULER-model: Ψ = 51;51;02
PETRIE-N.Side: Ψ = 51;50;40
ARRIS-model: Ψ = 51;50;39
RMP-model: Ψ = 51;50;34
PHI-model: Ψ = 51;49;38
CUBE-model: Ψ = 51;49;30

DNA - DOUBLE HELIX - 3 BASE CODONS
23 Chromosome Pairs .. “2” + “3”

``203 ≡ 𓍢𓍢▯▯▯

Three Squares -> Quaternions

$203 = 1^2 + 9^2 + 11^2$

$-3 = i^2 + j^2 + k^2$

Seven Squares -> Octonions

$203 = 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2$

$-7 = e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_5^2 + e_6^2 + e_7^2$

(″pmj:2023-03-21)